THE CATHOLIC UNIVERSITY OF AMERICA
DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING
CONSTRUCTION ENGINEERING AND MANAGEMENT PROGRAM

CONSTRUCTION INDUSTRY VALIDATION OF SCHEDULE PERFORMANCE MEASURE

Grant Number #07-16
Final Report
presented to the

Charles Pankow Foundation
and its Funding Partner
Construction Industry Institute

by

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Washington, DC 20064
June 1, 2019

TABLE OF CONTENTS

EXECUTIVE SUMMARY

Construction projects are risky, one-of-a-kind, complex endeavors. Each acts as its own 'temporary factory' established to create distinct facilities or structures. The construction industry in general suffers from time delays and cost overruns. Time delays are all too common in construction projects, and affect nearly 90% of large complex projects, with delays regularly exceeding the planned duration by 100 to 200%. Schedule performance, which measures the actual project completion time versus that planned, is a vital indicator of production efficiency, organizational maturity, competitiveness, and ultimately the ability to complete work in a timely manner. It is seldom considered after completing the work, and even more rarely evaluated on a comparative industry wide. This precipitated the National Research Council to conclude that "[c]onstruction firms do not have a single source of metrics for comparing the efficiency of their projects and processes, or for assessing their competitive position…and that there is no single, official index or measure for the productivity of the construction industry." Only safety performance currently has a predictive measure: The Experience Modification Rate (EMR) is utilized to determine the insurance premiums that individual contractors pay. It is defined as the moving average ratio of actual losses for the most recent completed two-years to expected losses within a three-year period and is widely accepted.

The purpose of this research was to build upon the theoretical foundations for a schedule performance measure (or index) developed under National Science Foundation (NSF) research. In specific, the purpose was to demonstrate a schedule performance measure's validity through an *a posteriori* application to completed construction project schedule networks, in fulfillment of the overarching NSF/CPF research purpose to:

> *". . . establish a 'living' schedule performance measure that will be comparable across project types and company sizes in the construction industry."*

The Capital Asset Pricing Model (CAPM) of financial portfolio theory describes the relationship between systematic risk and the expected return of individual stocks. Sharpe, Markowitz, and Miller shared the 1990 Alfred Nobel Memorial Prize in financial economics for this seminal work. Characterized as "one of the most important advances in financial economics," CAPM establishes a linear relationship between a stock portfolio's expected risk premium and the expected market risk premium, and by extension, the expected return for an individual stock. Extending the concept of Beta to construction project schedule systems (CPM networks) takes the form of a direct translation of variables. Disparate entities, herein defined as subcontractors, act within a complex decision-making process that is subject to constraints of whether a project will be on time or not. Like many individual assets comprise a financial market, so do many activities compose a project. As a performance measure, Schedule Beta takes on the following meaning for network schedule systems:

> *The aggregate measure of an individual schedule participant's project portfolio deviation/risk/performance correlation to the overall project portfolio, over a defined period of time and/or in total.*

In practice, implementation of Schedule Beta should replicate the approach of portfolio theory and be a continuous measure of an individual subcontractor's performance across all projects within which they worked. Like the Experience Modification Rate (EMR), which is a running average, performance could be measured over the work completed during the most recent two calendar years within the past three-year period.

Project schedule data have been obtained for an initial baseline project, expanded to a portfolio of projects, and ultimately over time reflected the final competed status of the project portfolio. Data that were collected underwent a detailed preparation to become usable for Schedule Beta calculations and analysis. They include Activity ID, Subcontractor Name, Subcontractor Discipline, Activity Description, Subcontractor As-Planned Duration, Subcontractor Actual Duration, Subcontractor Duration Delta, Project As-Planned Duration, Project Actual Duration, and Project Duration Delta. Data Requirements for Schedule Beta are: A minimum of two (2) activities for each subcontractor must be present for each project in which a subcontractor participated. It is concluded that the use of a single project, though representative of multiple phases, lacks sufficiency in data to properly depict an appropriate range for Schedule Beta. The longest meaningful activity durations are preferred. Dividing longer activity durations into multiple shorter activities is counter-productive.

Mathematically speaking, Schedule Beta can have positive or negative values. It is theoretically unlimited, but in practice is expected to have relatively small values around unity. Valid Schedule Betas were generated for fifty-two (52) subcontractors participating in at least two (2) of the eight (8) projects comprising the expanded dataset. Still, anomalies and outliers are found in the Schedule Beta calculations representing the expanded data set. Outlier Betas, those outside the bounds of the range from negative four to positive four ($-4 < \beta < 4$), are considered problematic to the point of impracticality. Scant variation across an array of individual subcontractor activities can result in an artificially high (absolute value) Schedule Beta due to a low Variance value, thereby skewing the Schedule Beta calculation due to its use as the denominator value – i.e., Schedule Beta = Correlation / Variance. Larger activity counts, in the case of the Final Dataset, in excess of one hundred (100) activities used for an individual subcontractor Schedule Beta calculation appear (anecdotally, as the project population and subcontractor contingent is not sufficient to depict a valid correlation) to yield better results. Participation in a larger pool of projects, herein in the case of the Final Dataset in excess of four (4) projects appears to yield a better Schedule Beta range, but the number of subcontractors in this category is insufficient to make a correlation beyond this generalization.

To properly advance the acceptance of implementation if Schedule Beta, it was decided to seek independent endorsement and standardization of the process and calculation. Research led to the American Society of Testing and Materials (ASTM) and their respective Committee E06 on Performance of Buildings and Subcommittee E06.81 on Building Economics. The research team is pleased to report that Schedule Beta has been accepted by Subcommittee E06.81 on Building Economics for development as a "Practice." An initial draft version for discussion will be considered during their October 2019 meeting.

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Acknowledgements

We would like acknowledge Mr. Mark Perniconi and Ms. Anne Ellis of the Charles Pankow Foundation (CPF) for their sponsorship of this research. The extension of the work would not have been possible without the time and effort of the Industry Advisory Panel, identified below, who played a critical role in the evaluation of the concept and data, and their support in the significant effort and timely review of the results.

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We also thank **Bozzuto Building Company** for full access to their project data and for their assistance in collection, evaluation, formatting, and dissemination of project data and performance and for describing their internal monitoring practices and systems.

Background

Construction projects are risky, one-of-a-kind, complex endeavors. Each acts as its own 'temporary factory' established to create distinct facilities or structures. They engage numerous subcontractors to facilitate construction, each of which interact with others while performing their many independent and dependent operations. All work is subject to the constraints of schedule, of budget, and of resource availability, in addition to project-specific physical conditions and the required functional specifications to which it must ultimately conform.

The construction industry in general suffers from time delays and cost overruns.[1] Cost overruns negatively affect project, contractor, and subcontractor performance and profitability, and give a poor image of the industry.[2] Time delays are all too common in construction projects, and affect nearly 90% of large complex projects, with delays regularly exceeding the planned duration by 100 to 200%.[3]

Schedule performance, which measures the actual project completion time versus that planned, is a vital indicator of production efficiency, organizational maturity, competitiveness, and ultimately the ability to complete work in a timely manner. It is seldom considered after completing the work, and even more rarely evaluated on a comparative industry wide. This is due to the price focus of the competitive bidding process (to acquire the next job) of the industry, and more importantly, because no unified measure of schedule performance – the actual versus the planned construction time – currently exists to benchmark schedule performance within the construction industry, or within the greater realm of operations research. While a plethora of sources focus on pricing and acquisition measures, limited information and methods exist to facilitate comparisons of the collective performance of contractors with respect to historic (and by extension also predictive) fulfillment of their contractual schedule commitments.

This precipitated the National Research Council[4] to conclude, "The U.S. construction industry does not have an industry-wide research agenda that identifies or prioritizes research areas with the most potential for improving its productivity, its competitiveness, or its efficiency," and to identify the obstacles to such an endeavor as the fact that, "[c]onstruction firms do not have a single source of metrics for comparing the efficiency of their projects and processes, or for assessing their competitive position…and that there is no single, official index or measure for the productivity of the construction industry." This resulted from a lack of adequate data, a lack of

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[1] Adam, A., Josephson, P., and Lindahl, G. (2017). Aggregation of factors causing cost overruns and time delays in large public construction projects: Trends and implications. *Engineering, Construction and Architectural Management*, 24(3), 393-406. https://doi.org/10.1108/ECAM-09-2015-0135.
[2] Toor, S. and Ogunlana, S. (2008). Problems causing delays in major construction projects in Thailand. *Construction Management and Economics*, 26(4), 395–408. https://doi.org/10.1080/0144619080190.
[3] Bhargava, A., Anastasopoulos, P., Labi, S., Sinha, K., and Mannering, F. (2010). "Three-stage least-squares analysis of time and cost overruns in construction contracts." *Journal of Construction Engineering and Management*, 136(11): 1207-1218.
[4] NRC. (2009). *Advancing the Competitiveness and Efficiency of the U.S. Construction Industry.* Report, Committee on Advancing the Productivity and Competitiveness of the U.S. Construction Industry Workshop, Board on Infrastructure and the Constructed Environment, Division on Engineering and Physical Sciences, National Research Council of the National Academies, Washington, District of Columbia: 114 pp.

consensus on appropriate measurement techniques, and a lack of consensus across the industry regarding the value of such measures.

Ultimately, the NRC concluded that the construction industry, its participants, and product (its individual projects) could benefit from further research, one element of which is to develop 'Project-level Measures' to understand:

> *". . . how an individual project compares with other, similar projects . . . in terms of total cost, schedule, cost changes, labor hours, and other factors. Such current measures are of greatest value to owners of multiple projects and to large contractors who are seeking to reduce the costs and delivery time of projects, to improve worker safety, or to initiate some other change in construction-related processes and practices*."

At present, only a simplistic schedule metric exists, the Schedule Performance Index (SPI) of the Earned Value Management (EVM) analysis system. With roots in project management on 1960s U.S. Government projects, SPI is defined as earned value (EV) divided by planned value (PV), but is only indicative in nature and has various well-known problems: This approach simply lists the percentage of total work or cites the 'value' that a schedule or budget has achieved. Conceptually it suffers from the problem that its values converge to 'one' (work completed) or 'zero' (work remaining) at the project end, so that the mere fact that a project was eventually finished makes a project appear successful – any problems that will have occurred are forgotten. This gives an overly optimistic, incomplete, and non-predictive view.

Another predictive measure within the construction industry is noteworthy for achieving such a performance measurement – that of safety: The Experience Modification Rate (EMR), which is unrelated to schedule measurement, is utilized to determine the insurance premiums that individual contractors pay. It is defined as the moving average ratio of actual losses for the most recent completed two-years to expected losses within a three-year period and is widely accepted. Similarly, examining other predictive measures outside the construction industry for extension and potential adaptation to solve this conundrum is where this research began.

Purpose

The purpose of this research was to build upon the theoretical foundations for a schedule performance measure (or index) developed under National Science Foundation (NSF) Research Grant Award No. 1536005, "Deriving a Standard Measure of Schedule Performance for Construction Project Management via Conceptual Analogy of Stock Performance versus Market." In specific, the purpose was to demonstrate a schedule performance measure's validity through an *a posteriori* application to completed construction project schedule networks. As conceived under the NSF grant, the Beta (β) component of the Capital Asset Pricing Model (CAPM) can be extended as a proxy to measure individual subcontractor performance within construction project network (critical path method – CPM) schedule systems, in fulfillment of the overarching NSF/CPF research purpose to:

". . . establish a 'living' schedule performance measure that will be comparable across project types and company sizes in the construction industry."

Theoretical Foundations

The Capital Asset Pricing Model (CAPM) of financial portfolio theory describes the relationship between systematic risk and the expected return of individual stocks. It is used within this research as a proxy, as a theoretical launching point, and as a model for the interaction of network schedule participants – namely subcontractors, and their individual activities that, together, form the intricacies and dependencies necessary to complete a construction project.

In its purest form, earnings are what drive the value or price of a stock.[5] However, due to the manner in which financial markets, and the stock market in particular, continuously change prices, they do not hold fast to this supposition. Rather, price differences are attributed to market conditions, market perception, and the behavior of market participants (investors). Similarly, in its purest form, the performance – the time duration – of a network schedule system are driven by the performance of its individual activities (the work of the subcontractor contingent in the case of a construction schedule). However, the manner in which individual activities progress, the changes in their durations, is dependent upon a multiplicity of schedule risks.

Characterized as "one of the most important advances in financial economics,"[6] CAPM was developed independently by Jack Treynor (1961, 1962), William Sharpe (1964), [7] John Lintner (1965), and Jan Mossin (1966), building on the earlier portfolio theory work of Harry Markowitz (1952). It establishes a linear relationship between a stock portfolio's expected risk premium and the expected market risk premium, and by extension, the expected return for an individual stock. It is based upon the assumption that the expected return on the market is equal to the risk-free rate plus some compensation (premium) for the inherent market risk. Sharpe, Markowitz, and Merton Miller (who built upon the theoretical work of Markowitz and Sharpe) shared the 1990 Alfred Nobel Memorial Prize in financial economics[8] for this seminal work.

Black, Jensen, and Scholes (1972)[9] defined this relationship as:

$E(R_i) = r_f + \beta_i [E(R_m) - r_f]$ where,

$E(R_i)$ = the expected return of a capital asset (an individual stock)
$E(R_m)$ = the expected return of the overall market
$E(R_m) - r_f$ = known as the "market premium" or the "risk premium," it is the difference

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[5] Maudlin, J. (2004). *Bull's Eye Investing: Targeting Real Returns in a Smoke and Mirrors Market*, John Wiley and Sons, Inc.: Hoboken, NJ.
[6] Ross, S., Westerfield, R., and Jaffe, J. (2005). *Corporate Finance*, 7th *Ed.*, McGraw-Hall Irwin: New York.
[7] Sharpe, W. (1964). "Capital asset prices: a theory of market equilibrium under risk." *Journal of Finance*, 19(3): 425 - 442.
[8] Royal Swedish Academy of Sciences (1990). "The Prize in Economics 1990 – Press Release." Nobelprize.org. <http://nobelprize.org/nobel_prizes/economics/laureates-/1990/press.html>, Accessed January 13, 2011.
[9] Black, F., Jensen, M. and Scholes, M. (1972). "The Capital Asset Pricing Model: Some Empirical Tests," ed. M. Jensen, Studies in the Theory of Capital Markets. Praeger Publishers: New York, NY.

between the expected market rate of return and the risk-free rate of return

r_f = the risk-free rate of interest such as interest arising from government bonds
R_i = the return of an individual asset
R_m = the return of the overall market
β_i, = beta is the sensitivity of the expected excess asset returns to the expected excess market returns, where:

$$\beta_i = \frac{Cov(R_i, R_m)}{Var(R_m)}$$

where,

$$Cov(R_i, R_m) = (R_i - \overline{R_i})(R_m - \overline{R_m})$$

and explicitly stated as:

$$Cov(R_i, R_m) = \sum_{n=1}^{N} \frac{(R_i - \overline{R_i})(R_m - \overline{R_m})}{N}$$

and,

$$Var(R_m) = (R_m - \overline{R_m})^2$$

or in explicit form as,

$$Var(R_m) = \sum_{n=1}^{N} \frac{(R_m - \overline{R_m})^2}{N}$$

where,

R_i = the return of an individual asset
$\overline{R_i}$ = the asset benchmark, commonly the previous day's asset return
R_m = the return of the overall market
$\overline{R_m}$ = the market benchmark, commonly the previous day's market return
N = the population size, the number of valuations being evaluated

From a pure mathematical perspective, calculating Beta can return a positive value or a negative value. Returning to the definition of Beta, the correlation of an individual asset (stock) to the overall market, a positive Beta indicates correlation in-concert with overall market direction, while a negative sign connotes an inverse correlation to the directional movement of the market. That is, a negative Beta means that the price of an individual stock moves opposite to the market.

Concept Definition

Disparate entities, herein defined as subcontractors, act within a complex decision-making process that is subject to constraints of whether a project will be on time or not. Like many individual assets comprise a financial market, so do many activities compose a project. The CAPM of portfolio theory offers a determinant for analogous behavior of subcontractors in projects and a potential measure of risk from uncertainties that may negatively impact network schedules.

Extending the concept of Beta to construction project schedule systems (CPM networks) takes the form of a direct translation of variables. Of particular concern is the definition of the benchmarks. The CAPM formula beta calculation identifies the benchmark for the rate of return for the entire market as $\overline{R_m}$. To extend this concept to network schedule systems, identification of an analogous components is required. As the benchmark for the rate of return of the entire market (in practice, the S&P 500 index) is the previous market rate of return over the given period being evaluated – commonly the previous day's market returns, the corresponding benchmark for network schedule systems is the as-planned durations. That is, d_i and $\overline{d_m}$ the corresponding benchmarks for as-planned activity duration and as-planned schedule duration respectively.

$E(R_i)$, the expected return of a capital asset, becomes:
    $E(d_i)$, the expected duration of an individual activity
$E(R_m)$, the expected return of the overall market, becomes:
    $E(d_m)$, the expected (as-planned) duration of the network schedule system
R_i, the return of an individual asset, becomes:
    d_i, the actual duration of an individual activity
R_m, the return of the overall market, becomes:
    d_m, the actual duration of the network schedule system

such that the CAPM equations translates as follows:

$$\beta_i = \frac{Cov(R_i, R_m)}{Var(R_m)} \quad \text{becomes} \quad \frac{Cov(d_i, d_m)}{Var(d_m)} = \beta_i$$

Substituting similarly to that of the explicit form for the calculation yields:

$$\beta_i = \frac{\sum(d_i - \overline{d_i})(d_m - \overline{d_m})}{\sum(d_m - \overline{d_m})^2}$$

where,

d_i = the as-built (actual) duration of an individual activity
$\overline{d_i}$ = the as-planned duration of an individual activity
d_m = the as-built (actual) duration of the network schedule system
$\overline{d_m}$ = the as-planned duration of the network schedule system

Schedule Beta

This is the seminal novel research-developed element and the basis for application to actual schedule data under this work. It was theoretically conceived under prior NSF research that the CAPM concept for measuring the risk of an individual stock versus the movement of the overall market can be extended to the performance of an individual activity within a project versus the overall project to form the performance measure identified as missing and sought by the NRC.

Based upon this, Schedule Beta takes on the following overall meaning for network schedule systems:

> ***The measure of individual schedule participant's deviation/risk/performance to the overall project schedule.***

As a performance measure, Schedule Beta takes on the following meaning for network schedule systems:

> ***The aggregate measure of an individual schedule participant's project portfolio deviation/risk/performance correlation to the overall project portfolio, over a defined period of time and/or in total.***

In practice, implementation of Schedule Beta should replicate the approach of portfolio theory and be a continuous measure of an individual subcontractor's performance across all projects within which they worked. Like the Experience Modification Rate (EMR), which is a running average, performance could be measured over the work completed during the most recent two calendar years within the past three-year period.

Research Data

Source and Confidentiality

Catholic University and Bozzuto Building Company, through the Industry Champion Mr. Kelly W. Wallace, have an existing Data Sharing Agreement on confidentiality and dissemination of data provided for use in research endeavors. Under the tenets of this agreement, project schedule data have been obtained for an initial baseline project, expanded to a portfolio of projects, and ultimately over time reflected the final competed status of the project portfolio.

Adhering to the confidentiality requirements of the agreement, a “blind” designation has been assigned to each project (in the form of an three (3) or four (4) letter acronym – i.e., “XXX”), which will yield an alpha numeric (acronym plus sequential number – i.e., “XXX01”) to blindly identify individual subcontractors. This becomes part of “Step 8 – Subcontractor Blind Designations” of the Data Formatting Process identified below.

Data Selection Criteria

Case Study Selection Criteria have been developed and vetted through the Industry Advisory Panel to select several real-world projects from Bozzuto Building Company upon which to test the new approach for schedule performance measurement, the following criteria have been used:

- *Location:* Projects should be in the same geographic (metropolitan) region for accessibility by the researchers, but more than one from same county or ward could also allow comparing within one environment / price level.
- *Type:* Projects should be of different type (or industry sector) if possible, e.g. residential / mixed residential-commercial / commercial / assisted living / high-rise versus low-rise / townhomes versus single family, renovation versus new construction, private versus institution owner, and similar. Horizontal projects (roads, bridges, tunnels, sewer / storm water, etc.) would be desirable from subcontractors or references. See ENR sectors.
- *Size:* Project should have somewhat different dollar ranges in built value or ‘small / medium / large’, e.g. <$10M, $10-50M, >$50M (per CMAA). As regions may have different cost levels, labor hours should also be known.
- *Delivery:* Projects should have a diversity of contract types if possible, e.g. DBB, DB(EPC), CM, CMGC, DBOM, etc.
- *Complexity:* Project schedules should have complexity (in terms of number of network activities and links between them) that ranges in hundreds to thousands. Schedule data should be stored in a common CPM computer format. Number and types of subcontractors and their share of work should be known, i.e. breakdown of self-performing versus subcontracting.
- *Duration:* Projects should have somewhat different duration ranges in schedule or ‘short / medium / long’, e.g. <1 year, 1-2 years, >2 years if possible.
- *Timing:* Projects should largely overlap with the two-year grant period of 2017-2018, but in somewhat different current stages or ‘early / medium / late’. Projects near inception would allow tracking much of actual progress. Realistically, tracking one full year of performance data would be ideal.
- *Performance:* Projects should have somewhat different performance in informal report as ‘green / yellow / red’ with actual differences from planned target. These should distinguish time performance from cost performance and inform about percent complete, milestones, and similar relevant data.

- *Permission:* Bozzuto needs to give written data use permission for research purposes for selected projects and not be confidential or proprietary (e.g. secret government agency facilities or restricted data access).
- *Staffing:* Project managers (and potentially also subcontractor representatives) need to be available for being interviewed about their schedule planning and control approaches, as well as their forecasted and actual past, present, and future project performance.
- *Common:* Projects should be somewhat archetypical in nature to be somewhat representative and generalizable to other regions within and beyond the U.S.

Ultimately, a list of twenty-two (22) projects was submitted for consideration by Bozzuto Building Company through Mr. Wallace. An initial project was identified to be used as a Case Study to demonstrate the calculation, application, and meaning of Schedule Beta.

Initial Data Set – Baseline Project Definition

An initial project was selected to use as a Case Study for Schedule Beta, blindly described as "MSM.". The project was selected in accordance with the above-defined criteria, and in specific, due to its completed status and that it contains multiple definable phases representative of individual projects – thereby constituting a 'portfolio' of projects.

MSM is characterized as follows: A mixed-use urban revitalization development of five blocks of buildings in a metropolitan Washington, DC neighborhood. It consists of approximately 720 residential units, 45 townhomes, 83,000 square feet of street-level retail, 15,000 square feet of artist studio space, a 3,000 square-foot community arts center, and 850 parking spaces, was completed between 2011 and 2014, with a constructed value in excess of $100 million.

The dataset provided consisted of the baseline ("as-planned") schedule plus monthly schedule updates for each block over the 27-month long duration, the final (as-constructed) schedule, plus all invoices, subcontracts, and plan drawings.

The unique nature of this development makes it an ideal data source for this research:

- Each block is a separately organized project with its own project manager, and was ultimately within the work overseen by then-construction executive and Industry Champion, Mr. Wallace;
- All blocks have been performed by the same subcontractor contingent in the same environment and timeframe;
- Sufficient variety exists to compare performance of different means and methods. Some underground parking is adjacent to a metro culvert, other is not. Cast-in-place concrete reaches to first floor retail areas, upper floors have a wood frame structure. The exterior finish is both brick and siding as façade.

Expanded Data Set

It was determined (as described below in the "Interim Analysis of Schedule Beta" subsection of the section titled "Initial Case Study (MSM) results") that additional project datasets were necessary to properly calculate and depict Schedule Beta. In accordance with the criteria and process identified in the section titled "Data Selection Criteria," seven (7) additional projects were selected under the same data sharing agreement and confidentiality conditions, and their data was evaluated and formatted similarly. They are as follows, yielding a "project portfolio," including the initial Case Study project, MSM:

ANT: A nine-story tall 292-apartment high-rise community including 20,000 square feet of retail and 449 parking spaces, with a rooftop lounge, sundeck, fitness center, and swimming pool in a Washington, DC suburb.

Project Value:	$75 million
As-Planned Duration:	883 days
Actual Duration:	760 days
Completion Status:	Completed 08/2017

ANTH: A four-story wood framed 52-apartment building on a concrete podium with retail at the ground floor constructed as phase two of the ANT project.

Project Value:	$12 million
As-Planned Duration:	408 days
Actual Duration:	439 days
Completion Status:	Competed 08/2018

CPR: A market-rate multi-family high rise 179-apartment building with a roof top terrace and ground floor retail space located walking distance from a sports complex in metropolitan Washington, DC.

Project Value:	$43 million
As-Planned Duration:	980 days
Actual Duration:	700 days
Completion Status:	Completed 03/2019

MPLV: Four (4) story 116,000 square foot wood framed 114-unit affordable housing apartment building over a one-story podium, two levels (52,000 square feet) of below-grade parking, and 15,000 square feet of ground-level retail space.

Project Value: $35 million
As-Planned Duration: 464-days
Actual Duration: 496-days
Completion Status: Completed 11/2018

MSM – Data Developed as the original Case Study Project: A mixed-use urban revitalization development, consisting of five (5) city blocks in metropolitan Washington, DC. The project includes approximately 720 residential units, 45 townhomes, 83,000 square feet of street-level retail, 15,000 square feet of artist studio space, a 3,000 square-foot community arts center, with 850 structured parking spaces.

Project Value:	$100 million
As-Planned Duration:	911 days
Actual Duration:	911 days
Completion Status:	Completed 10/2014

SGRV: Transit-oriented and mixed-use development featuring residential and retail space, located in proximity of the Washington, DC Metro Red Line. The project includes 333 wood-framed apartment units, 13,100 square feet of indoor amenity space, 17,300 square feet of ground-floor retail, and a 441-space precast parking garage. The project is designed and constructed to be LEED Silver Certified.

Project Value:	$52 million
As-Planned Duration:	687 days
Actual Duration:	680 days
Completion Status:	Completed 10/2017

SME: Seven-story 202-unit mixed-use affordable community with 31 family housing units and 151 single adult housing units, consisting of three (3) stories of concrete podium and four (4) stories of metal-frame bearing on composite steel and concrete floor assembly. The center includes over 105,000 square feet of below-grade parking (three levels, 188 spaces) and in excess of 110,000 square feet of commercial retail space, including a medical offices. The project is designed and constructed to be LEED Silver Certified.

Project Value:	$63 million
As-Planned Duration:	779 days
Actual Duration:	740 days
Completion Status:	Completed 06/2018

WLTN: Three (3) four-story slab-on-grade garden-style apartment buildings, each with 72 market-rate units. Buildings have exterior breeze-ways, no elevators.

Project Value:	$28 million ($9.3 million per building)
As-Planned Duration:	855 days
Actual Duration:	805 days
Completion Status:	Completed 09/2018

Final (Complete) Data Set

Periodic updates were received during the course of the research work for projects not yet complete, culminating in late Spring 2019 with completion of the last project used. The project portfolio used as the final dataset consists of the projects identified in Table 1: Project Identification, Durations, and Status.

Table 1: Project Identification, Durations, and Status

Project Acronym	As-Planned Duration	Actual Duration	Subcontractors	Activities
ANT	883 days	760 days	48	1,095
ANTH	408 days	439 days	30	552
CPR	980 days	700 days	33	831
MPLV	464 days	496 days	36	1,045
MSM	911 days	911 days	48	964
SGRV	687 days	680 days	46	1,607
SME	779 days	740 days	41	976
WLTN	855 days	805 days	33	1,231

Data Formatting – Generic Process

Data that were collected underwent a detailed preparation to become usable for Schedule Beta calculations and analysis. Raw schedule data, extracted by the Industry Partner from the common Oracle Primavera P6 software required conversion into Microsoft Project file format and then into numeric representation within multiple Microsoft Excel spreadsheets to be usable for mathematical operation and Schedule Beta calculation. Raw data consist of the baseline as-planned project schedule, ongoing monthly updates, and the final completed project as-built data.

Required input data to calculate Schedule Beta consist of six elements:

1. Activity identifier (numeric label)
2. Subcontractor name
3. Subcontractor blind designation
4. Activity description
5. As-planned duration, and
6. As-built duration

Superfluous data were discarded during the preparation process. This process has two phases; *Extraction and Formatting* (steps 1 and 2) and *Verification and Validation* (steps 3-8). It was vetted with the Industry Advisory Panel, tested, refined, and applied as follows:

Step 1a – Raw Data Preparation: After extracting CPM updates into individual spreadsheets representing the baseline schedule, monthly updates, and final schedules. Non-activity specific elements (section titles and headings, non-individual phase durations (i.e. hammocks), and milestone events, i.e. all horizontal rows of secondary information were removed.

Step 1b – Beta-Specific Column Formatting: Removal of non-duration data such as predecessor / successor identification, and comments, i.e. all vertical columns of secondary information are removed. Each spreadsheet contains six data elements with the blind designation.

Step 2 – Alphanumeric Naming and Sort: Assignment of a letter designation to each phase and/or project component to differentiate participant activities and facilitate calculation individual Schedule Betas.

Step 3a – Matching As-Planned to As-Built (First to Last): Data were reviewed to determine if any discrepancies exist between spreadsheets, notably the activity count. Initial (as-planned) scheduled typically contain fewer individual activities than the final, which requires additional formatting:

- Compare Schedules: Compare the baseline schedule and each monthly update to the final schedule and insert blank rows within the Microsoft Excel worksheets wherever activities are not present in a corresponding update. This will create identical numbers of rows in all spreadsheets.
- To identify any errors between updates, both activity identifiers are matched row by row with a LOOKUP command. The order of activities is checked. This gives four possible outcomes:

Check ID	*Check Order*
MATCH	TRUE (correct)
MATCH	FALSE (incorrect)
NOT	TRUE (incorrect)
NOT	FALSE (incorrect)

The only acceptable outcome is MATCH-TRUE. Other cases require individual review to modify it by inserting or deleting blank rows or shifting a full row until the correct outcome.

Change Orders: Should the final project and/or individual activity durations include time extensions by way of Change Orders, the baseline as-planned duration is to be modified to reflect the impact of the change orders.

Step 4b – Origin of Activities: Given that the activity count grows over successive updates, the first occurrence of added activities must to be identified to determine their as-planned durations. It can be expected that few activities exist in their original form from the baseline schedule through updates to the final as-built schedule, because activities may be split, eliminated, changed, or added as schedules are refined during execution. Using another LOOKUP command identify such 'stable' activities. Overall, activities may appear in all schedule updates or in as few as two (the last update and the final as-built schedule). To trace this intricate evolutionary history of unique activities, the following approach is devised:

- A LOOKUP command records occurrences across all updates with three possible outcomes:

  If $n = 1$, then no further entries exist in updates and the activity is deleted from the data.
  If $n > 1$, then it is listed multiple times; its first and the last valid values can be extracted.
  If $n < 1$, then an error exists in the data, as any activity must be mentioned at least once.

Step 4c – Adding Missing Durations: Determine the as-planned and as-built durations for those activities that are identified in the previous step, moving sequentially through successive updates.

Step 5 – Eliminating Duplicate Activities: Identify and correct any duplications in the dataset:

1. If IDs are repeated but descriptions differ, separate them into two different activities.
2. If IDs differ but descriptions are repeated, separate them into two different activities.
3. Identify were a unique activity first and last appears in schedule updates. Record the first and last occurrences as respective as-planned and as-built durations for analysis.
4. Repeat previous step for subsequent activities. Label multiple occurrences as needed.

Step 6 – Compiling Formatted Data: Create an as-built schedule spreadsheet by assembling the newly extracted data from the previous steps and name the file "Project [Name] Schedule Data".

Step 7 – Missing Participant Names: Review the formatted data with the Industry Partner versus an existing project participant list and assign to any activities that do not list their subcontractor.

Step 8 – Subcontractor Blind Designations: Compile subcontractor list into tabular form and assign alphanumeric designations for anonymity in publications per confidentiality requirements of Data Sharing Agreement.

A comparison of the beginning "formatted" data to the "calculation" necessary data resulting from the Extraction and Formatting, and Verification and Validation process is:

Formatted Data	Calculation Data
Column 1: Activity ID	Column 1: Activity ID
Column 2: Subcontractor Name	Column 2: Subcontractor Name
Column 3: Subcontractor Discipline	Column 3: Subcontractor Discipline
Column 4: Activity Description	Column 4: Activity Description
Column 5: As-Planned Duration	Column 5: Subcontractor As-Planned Duration
Column 6: Actual Duration	Column 6: Subcontractor Actual Duration
	Column 7: Subcontractor Duration Delta
	Column 8: Project As-Planned Duration
	Column 9: Project Actual Duration
	Column 10: Project Duration Delta

Results suitable for publication from this process are depicted in Table 2: Sample Formatted Data.

Table 2: Sample Formatted Data

Activity ID	Subcontractor Name	Discipline	Blind Designation	Description	Duration	
					As-Planned	As-Built
Project Segment 1A					**403**	**410**
C1140	"Redacted"	"Redacted"	Contractor CCC03	Tub Surrounds	6	6
C1150	"Redacted"	"Redacted"	Contractor CCC03	Install Flooring	6	6
C1230	"Redacted"	"Redacted"	Contractor CCC03	Install Carpet	4	4

Schedule Beta Calculation

Calculation Methodology

To calculate Schedule Beta per the following equation (as initially derived above in the section titled "Concept Definition"),

$$\beta_i = \frac{\text{Cov}\,(d_i, d_m)}{\text{Var}\,(d_m)}$$

and

$$\beta_i = \frac{\sum (d_i - \overline{d_i})(d_m - \overline{d_m})}{\sum (d_m - \overline{d_m})^2}$$

Microsoft Excel functions are implemented to determine activity and project counts and the quotient of statistical covariance and variance for each subcontractor:

*Number of Activities:**	*=COUNT(D_i;D_i)*
*Number of Projects: **	*=COUNT(D_m; D_m)*
Statistical Covariance:	*=COVAR(D_i: D_i, D_m: D_m)*
Statistical Variance:	*=VAR(D_i: D_i)*
Schedule Beta:	*=COVAR(D_i: D_i,Y D_m: D_m) / VAR(D_i: D_i)*

where,

D_i = the individual activity duration delta ("as-built" minus "as-planned")
D_m = the overall project duration delta ("as-built" minus "as-planned")
* = for statistical validation purposes, not used in Schedule Beta calculation

To facilitate Schedule Beta calculation, additional elements are needed, namely the duration deviation from the as-planned benchmark for each activity and its respective project. This is accomplished calculating the “as-built” duration minus “as-planned” duration. These differences become the inputs for the covariance and variance functions.

An example calculation data spreadsheet for a hypothetical Subcontractor C17 is depicted in Table 3: Beta Schedule Performance Measure Calculation Table – Subcontractor 17:

Table 3: Beta Schedule Performance Measure Calculation Table – Subcontractor 17

Activity ID	Name	Blind Designation	Description	Duration					
				Activity			Project		
				As-Planned	As-Built	Delta	As-Planned	As-Built	Delta
C01010	*Redacted*	C17	Layout	5	5	0	403	410	7
E00010	*Redacted*	C17	Layout	1	1	0	403	410	7
D001020	*Redacted*	C17	Layout	5	5	0	218	257	39
B01010	*Redacted*	C17	Layout	5	5	0	461	480	19
A01010	*Redacted*	C17	Layout	5	5	0	510	566	56
				Variance		**0**	**Covariance**		**0**
				Activity Count		**5**	**Beta**		**?**
				Project Count		**4**			

Theoretical Meaning of Schedule Beta

Mathematically speaking, Schedule Beta can have positive or negative values. It is theoretically unlimited, but in practice is expected to have relatively small values around unity. Of particular interest are the values zero and one that divide its continuous spectrum of values into the following ranges:

- $\beta > 0$ means that the subcontractor performance tends to move same as projects on which they work;
- $\beta < 0$ means that the subcontractor performance tends to move in opposite direction as their projects and while it may occur in small datasets, it is considered unrealistic over longer timeframes;
- $\beta > 1$ or $\beta < -1$ means that the subcontractor incurs stronger deviations than project itself and is risky;
- $\beta = 1$ means that that no distinction between the subcontractor and project performance can be made;
- $\beta = 0$ means that activities move independently from their projects, which is unlikely, or that planned and actual values are identical, which would cause impermissible division by zero;
- $0 < \beta < 1$ or $-1 < \beta < 0$ means that the subcontractor has less deviations than project and is less risky.

Note that these ranges overlap. These interpretations mirror the common explanations of risk and return behavior of stocks relative to their markets within the financial literature.[10]

Initial Case Study (MSM) Results

Schedule Beta Calculation Approach

The dataset for the Case Study Project, "MSM" was aggregated into its respective parts – three (3) city blocks and five (5) individual "project segments" as follows:

- Block 1: Two (2) adjacent buildings containing artist studios, townhouses, and an arts center.
- Block 2: A single zero lot line building containing subterranean parking, street-level commercial/retain space, and residential units.
- Block 3: A single zero lot line multi-courtyard "donut" building containing subterranean parking, street-level commercial/retain space, and residential units.
- Street and Utility Work: Off-site work within the Right-of-Way

An examination of the formatted data led the research team to consider two options for calculating Schedule Beta:

> "Per-Project" Calculation: Wherein the as-planned durations and as-built durations for all individual schedule activities for each subcontractor for each project are combined into a single value, from which their respective deltas ("as-built" minus "as-planned") are determined.
>
> "Per-Activity" Calculation: Wherein the as-planned durations and as-built durations for each individual schedule activity for each subcontractor for each project remain independent, from which their respective deltas ("as-built" minus "as-planned") are determined. This oprtion has been used herein.

Discussion of Resulting Schedule Betas

Subcontractors with valid Schedule Beta calculations values are summarized in Table 4: Schedule Beta Calculation Summary. Numerous Schedule Beta calculation were found to have anomalies necessitating their exclusion from consideration. This was due to:

1. The subcontractor participating in only one (1) component of the Case Study Project. This made calculation of Schedule Beta impossible, as the statistical variance function requires more than one (1) value for calculation (applicable to the "Per-Project calculation). That is, there can be no "variation" across a project

[10] Gatfaoui, H. (2010). "Capital asset pricing model." In *Encyclopedia of Quantitative Finance*, ed. Cont, R., John Wiley and Sons, Hoboken, New Jersey.

"population" when the subcontractor participates in a single project. Similarly, when a subcontractor's participation is single array of values, the Covariance is by definition zero, rendering calculation of mathematically impossible (applicable to the Per-Activity calculation).

2. Variance Calculation Equal to Zero: The Variance is by definition zero when all values used in its calculation (the respective "Deltas") are zero. This occurs when all work, irrespective of calculation method defined above, is completed as-planned, resulting in no delta between as-planned and as-build performance.
3. Schedule Beta Equal to Zero: Schedule Beta may return a zero value. This occurs when the Covariance function uses the same value for all comparisons – due to the activities of a subcontractor being relegated to a single project.

Table 4: Schedule Beta Calculation Summary

Subcontractor	**"Per Project" Calculation**			**"Per Activity" Calculation**		
Designation	**Number of Projects**	**Variance**	**Beta**	**Number of Activities**	**Variance**	**Beta**
MSM02	4	-44.00	**-0.97**	85	-1.79	**-2.39**
MSM03	2	-12.50	**-11.40**	4	-2.25	**-8.50**
MSM04	4	-6.25	**-2.25**	4	-6.25	**-2.25**
MSM06	2	-544.50	**0.56**	4	-272.25	**0.28**
MSM07	4	315.58	**0.83**	91	3.57	**3.19**
MSM09	4	-54.25	**-2.05**	30	-4.31	**-4.92**
MSM10	2	8.00	**8.13**	7	1.95	**2.72**
MSM16	4	-56.92	**-1.61**	214	-0.74	**-2.69**
MSM18	4	-75.00	**-0.73**	52	-7.15	**-0.17**
MSM19	4	-2.25	**-7.75**	24	-0.20	**-15.26**
MSM20	4	6.25	**-4.65**	13	1.92	**-2.86**
MSM22	4	-45.33	**-1.70**	26	-3.37	**-3.64**
MSM26	4	-4.00	**6.44**	23	-0.70	**5.23**
MSM28	4	9.00	**4.29**	43	0.60	**2.71**
MSM35	3	-5.33	**-2.08**	18	-2.30	**-1.27**
MSM39	2	-450	**0.62**	2	-450	**0.62**

Observations of MSM Subcontractor Contingent Schedule Betas

1. The range of Schedule Beta, -15.26 to 8.13 is excessively wide. Loosely characterized as fifteen fold and eight fold schedule performance magnitude correlation to that of the project

portfolio. Anecdotally determined to be the result of scant variance from as-planned duration, yielding a low Variance.
2. The quantity of negative Schedule Betas is disproportionate. Anecdotally attributed to the relatively successful performance of the overall Case Study Project and its subcomponents.
3. The order of magnitude of Schedule Beta is relatively consistent between the two methodologies, though differences are noted. It can be anecdotally concluded that large quantities of activities used to calculate Schedule Beta yields tighter and/or more statistically valid results.

Interim Analysis of Schedule Beta

Based upon the disparate Schedule Beta values generated, it is concluded that the use of a single project, though representative of multiple phases, lacks sufficiency in data to properly depict an appropriate range for Schedule Beta. It was recommended that the dataset for extended to include the additional projects made available by Bozutto Construction Company and the Industry Advisor, thereby translating the work of this research team from a single Case Study Project approach to more closely represent a financial market by utilizing multiple distinctive projects.

Thus, the following characterizations can be made:

1. The Schedule Performance Measure, "Schedule Beta," mathematics can be translated from its origin in the Financial Beta from the Capital Asset Pricing Model (CAPM), and are sound,
2. Calculation of Schedule Beta has limiting factors, such as:
   a. Deviation from as-planned is required for the "Variance" component to exceed zero, else it results division by zero and an undefined Schedule Beta,
   b. Schedule Beta equal to zero is possible and results from the "Covariance" calculation using the same values for all "project" duration delta values,
   c. Multiple activities must be represented in each independent project for Schedule Beta,
3. Appropriate data are available from completed construction projects in the form of as-planned activity durations versus actual durations,
4. Data and analysis from the MSM Case Study Project proved insufficient to "prove" the Schedule Beta concept.

From this, the expanded project dataset results in the following observations about the development of Schedule Beta:

Data Requirements for Schedule Beta are:
- A minimum of two (2) activities for each subcontractor must be present for each project in which a subcontractor participated.
- The longest meaningful activity durations are preferred. Dividing longer activity durations into multiple shorter activities is counter-productive.
- A robust dataset for Schedule Beta include for each completed project:

- Project Name
- Activity Identification (ID Number)
- Activity Name
- Activity Description
- Activity As-Planned Duration and corresponding Total Project As-Planned Duration
- Activity Actual Duration and corresponding Total Project Actual Duration

Calculation Requirements for Schedule Beta are:

- All project work must be complete to be used in the calculation of Schedule Beta.
- A minimum of two (2) projects must be used to calculate a Schedule Beta.
- Variance must exist across the collection of activities for a minimum of two projects of the cadre forming the dataset for the calculation of Schedule Beta to be possible.

Further explanation of Schedule Beta calculation, its behavior and idiosyncrasies are presented in Appendix A: Behavior of Schedule Beta, Appendix B: Population versus Sample Size Factor, and Appendix C: Sensitivity of Schedule Beta – Balanced / Unbalanced Activity Splitting and Addition.

Expanded Project Population Results

Schedule Beta Calculation Approach

Calculation of Schedule Beta across a mutually-exclusive project population (unlike the Case Study Project where it was determined that the subcontractor and schedule activity population was insufficiently diverse to generate valid Schedule Betas) were calculated following the "Per-Activity" calculation variation determined to be the most accurate and, subsequently, the desired methodology. It provided the tightest Schedule Betas range and, as discussed in the Appendices, fulfilled the need for activity quantity and diversity required by the Variance and Correlation statistical functions. It is a conclusion of this research that the Per-Activity calculation variant be the definition of Schedule Beta (and will be so defined in the forthcoming American Society of Testing and Materials, ASTM, standard).

Subcontractor and the corresponding project performance data completing the ten (10) data columns were aggregated from projects in which there was participation, and the valid Schedule Betas (those meeting the Data Requirements as defined and found in the section titled "Interim Analysis of Schedule Beta") were generated for fifty-two (52) subcontractors participating in at least two (2) of the eight (8) projects comprising the expanded dataset (the Case Study "MSM" project plus the seven (7) additional completed projects supplied by the Industry Champion through Bozzuto Building Company). Results are presented in Table 5: Final Dataset Schedule Beta Calculation Summary and graphically in Figure 1: Scatter Plot of Final Dataset Schedule Betas, along with their statistical representation in Table 6: Final Dataset Schedule Beta Statistical Summary.

Table 5: Final Dataset Schedule Beta Statistical Summary

Project Count	52
Positive	29
High	14.91
Negative	23
Low	-14.21
Mean	0.17
Mean Positive	3.00
Mean Negative	-3.40

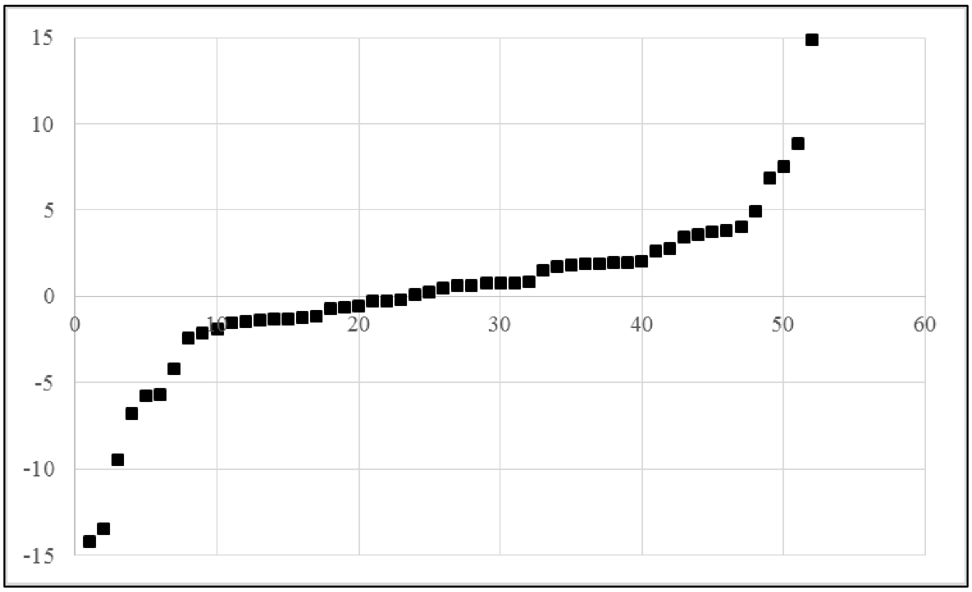

Figure 1: Scatter Plot of Final Dataset Schedule Betas

Table 6: Final Dataset Schedule Beta Calculation Summary

Blind Designation	Activity Count	Project Participation	Schedule Beta
ANT02	123	3	**-2.13**
ANT04	29	3	**-0.66**
ANT07	119	2	**0.75**
ANT12	16	2	**4.90**
ANT14	59	3	**1.77**
ANT15	29	2	**0.65**

ANT17	24	2	**0.49**
ANT20	84	3	**-1.30**
ANT21	58	3	**-1.24**
ANT25	24	2	**-1.38**
ANT29	12	3	**1.73**
ANT36	61	2	**2.01**
ANTH05	70	2	**0.58**
ANTH10	46	3	**1.97**
CPR09	65	2	**6.84**
MSM05	80	2	**0.08**
MSM06	64	4	**1.85**
MSM07	834	6	**-0.28**
MSM09	363	4	**-1.91**
MSM10	29	3	**4.04**
MSM14	4	2	**-4.19**
MSM16	442	2	**0.81**
MSM17	7	2	**-9.50**
MSM26	31	2	**1.97**
MSM27	119	5	**-0.72**
MSM29	53	3	**-1.13**
MSM34	130	3	**0.79**
MSM35	71	4	**1.89**
MSM36	150	5	**2.75**
MSM37	2	2	**-1.30**
MSM39	51	4	**1.51**
SGRV02	39	2	**-2.42**
SGRV03	56	2	**-1.57**
SGRV07	3	2	**-1.43**
SGRV09	16	4	**3.73**
SGRV10	152	4	**3.61**
SGRV17	60	2	**-5.75**
SGRV18	4	2	**3.80**
SGRV22	71	2	**-0.19**
SGRV25	26	3	**-6.81**
SGRV26	7	2	**7.50**
SGRV28	26	2	**-13.50**

SGRV32	19	2	**-5.68**
SME01	26	2	**14.91**
SME03	18	3	**-14.21**
SME13	98	3	**2.62**
SME14	134	3	**0.77**
SME16	97	3	**-0.58**
SME18	7	2	**3.43**
SME19	28	2	**-0.30**
SME25	89	2	**0.27**
WLTN19	25	2	**8.88**

Discussion of Resulting Schedule Betas

Calculation of Schedule Beta following the requirements emanating from the Cast Study Project analysis eliminated mathematical impossibilities, by setting minimum activity and project participation requirements,. Still, anomalies and outliers are found in the Schedule Beta calculations representing the expanded data set.

1. Range of Schedule Betas: Outlier Betas, those outside the bounds of the range from negative four to positive four ($-4 < \beta < 4$), are considered problematic to the point of impracticality. This is posited as by definition, a Schedule Beta is the correlation to project schedule movement wherein a Schedule Beta of 4 connotes subcontractor activity of four times that of the project. Simply characterized, project schedule delay (increased duration) of one (1) week would predict a four (4) week increase for a subcontractor with a Schedule Beta equal to 4. Take to the extreme, when negatively correlated, a subcontractor with a Schedule Beta of -4 could expect a four (4) month delay when the project finishes one (1) month ahead of as-planned schedule.

   Such Schedule Beta Outliers, herein identified in Table 7: Final Dataset Schedule Beta Calculation Outliers and found within the larger Figure 2: Scatter Plot of Final Dataset Schedule Betas versus Activity Count, result from the following:

Table 7: Final Dataset Schedule Beta Calculation Outliers

Blind Designation	Activity Count	Project Participation	Schedule Beta
ANT12	16	2	**4.90**
CPR09	65	2	**6.84**
MSM10	29	3	**4.04**
MSM14	4	2	**-4.19**

MSM17	7	2	**-9.50**
SGRV17	60	2	**-5.75**
SGRV25	26	3	**-6.81**
SGRV26	7	2	**7.50**
SGRV28	26	2	**-13.50**
SGRV32	19	2	**-5.68**
SME01	26	2	**14.91**
SME03	18	3	**-14.21**
WLTN19	25	2	**8.88**

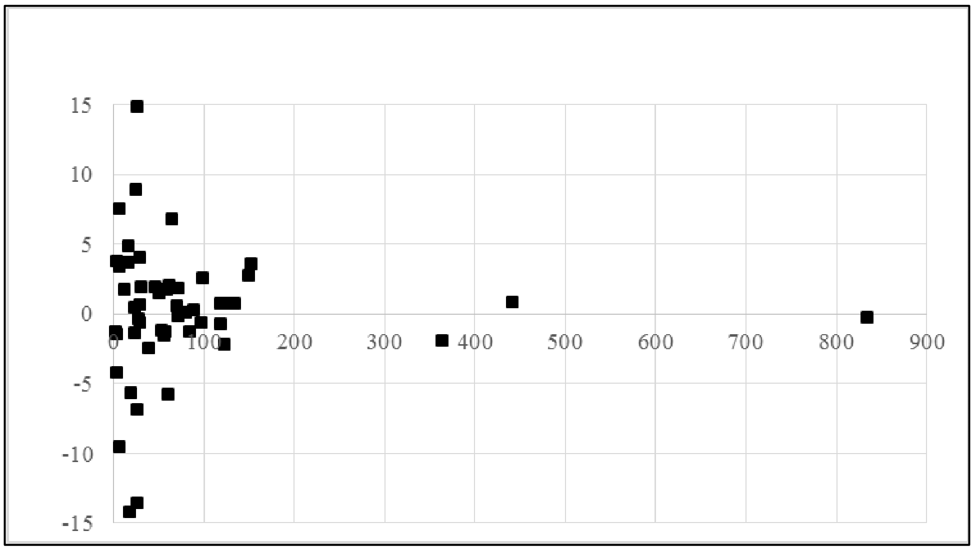

Figure 2: Scatter Plot of Final Dataset Schedule Betas versus Activity Count

For example, subcontractor SME01's Schedule Beta equal to 14.91 resulted from uniform variation to as-planned duration against consistently short durations: Twenty-one (21)five-day as-planned activities with single day as-built durations coupled with five (5) other activities, four (4) of which did not vary from as-planned values – as depicted in Table 8: Subcontractor SME01 Schedule Beta Activity Data. This yielded an artificially low Variance (denominator) value, thereby yielding an impractical Schedule Beta.

Table 8: Subcontractor SME01 Schedule Beta Activity Data

Activity ID	As-Planned Duration	As-Built Duration	Performance Delta	Schedule Performance Delta
SO1021	5	1	**-4**	-39
SO1030	5	1	**-4**	-39
SO1058	5	1	**-4**	-39

SO1067	5	1	**-4**	-39
SO1094	5	1	**-4**	-39
SO1103	5	1	**-4**	-39
SO1131	5	1	**-4**	-39
SO1140	5	1	**-4**	-39
SO1167	5	1	**-4**	-39
SO1176	5	1	**-4**	-39
SO1203	5	1	**-4**	-39
SO1212	5	1	**-4**	-39
SO1239	5	1	**-4**	-39
SO1248	5	1	**-4**	-39
SO742	5	1	**-4**	-39
SO840	5	1	**-4**	-39
SO870	5	1	**-4**	-39
SO871	5	1	**-4**	-39
SO899	5	1	**-4**	-39
SO984	5	1	**-4**	-39
SO993	5	1	**-4**	-39
MW770	3	5	**2**	32
MW819	3	3	**0**	32
MW864	4	4	**0**	32
MW907	6	6	**0**	32
MW948	4	4	**0**	32

2. Quantity of Activities: Ten (10) subcontractors have in excess of one hundred (100) individual activities use in the calculation of Schedule Beta. Table 9: Final Dataset Subcontractors with More than 100 Activities and Figure 3: Scatter Plot of Final Dataset Schedule Betas versus Activity Count in Excess of 100 depict these subcontractors.

Table 9: Final Dataset Subcontractors with More than 100 Activities

Blind Designation	Activity Count	Project Participation	Schedule Beta
ANT02	**123**	3	-2.13
ANT07	**119**	2	0.75
MSM07	**834**	6	-0.28
MSM09	**363**	4	-1.91

MSM16	**442**	2	0.81
MSM27	**119**	5	-0.72
MSM34	**130**	3	0.79
MSM36	**150**	5	2.75
SGRV10	**152**	4	3.61
SME14	**134**	3	0.77

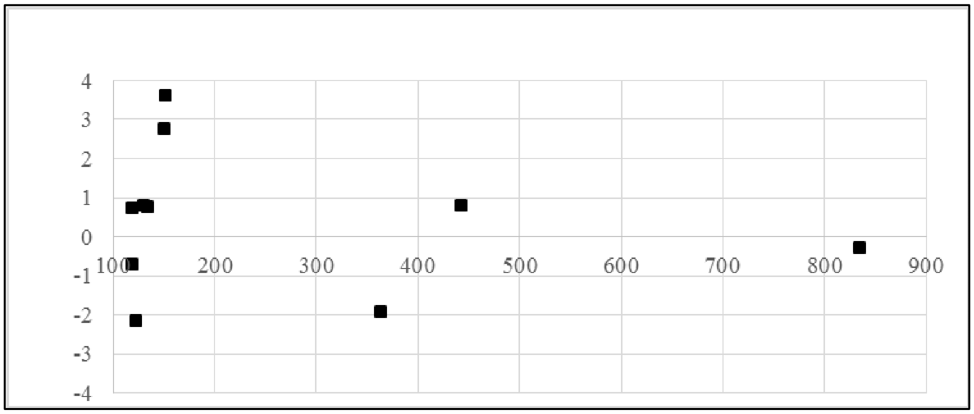

Figure 3: Scatter Plot of Final Dataset Schedule Betas versus Activity Count in Excess of 100

Based on the ten data points for activity counts in excess of 100, the statistical evaluation of the Schedule Betas outperforms that of the overall population. Both positive and negative Schedule Betas exist, but the range is tighter, from -2.13 to 3.61, thereby eliminating outliers. Table 10: Final Dataset Schedule Beta Statistical Summary for Activity Counts in Excess of 100 depicts these statistics.

Table 10: Final Dataset Schedule Beta Statistical Summary for Activity Counts in Excess of 100

Project Count	10
Positive	6
High	3.61
Negative	4
Low	-2.13
Mean	0.44
Mean Positive	1.58
Mean Negative	-1.26

Conversely, when Schedule Betas are considered for subcontractors with less than 150 activities, a better dispersion of Schedule Betas results, and is depicted in Figure 4: Scatter Plot of Final Dataset Schedule Betas .versus Activity Count in Below 150

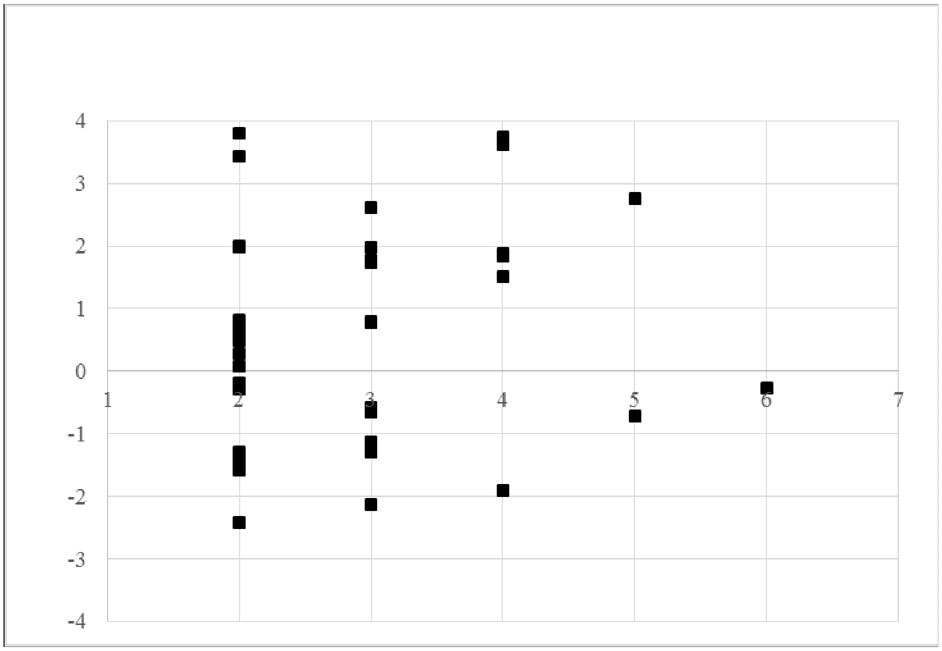

Figure 4: Scatter Plot of Final Dataset Schedule Betas versus Activity Count in Below 150

3. Project Count: Few subcontractors within the Final Dataset participate in four (4) or more projects, and only three (3) participating in five (5) or more (two (2) in five (5) projects and one (1) in six (6) projects.. Table 11: Final Dataset Subcontractors Participating in More than Four Projects and Figure 5: Scatter Plot of Final Dataset Subcontractors Participating in More than Four Projects depict these outliers – considered to be outliers, as the collection of nine (9) projects represents less than twenty (20) percent of valid Schedule Beta calculations. It appears that increased project counts also yields tighter Schedule Betas, there are too few data points to make such a generalization.

Table 11: Final Dataset Subcontractors Participating in More than Four Projects

Blind Designation	Activity Count	Project Participation	Schedule Beta
MSM07	834	6	-0.28
MSM36	150	5	2.75
MSM27	119	5	-0.72
MSM09	363	4	-1.91
SGRV10	152	4	3.61
MSM35	71	4	1.89

MSM06	64	4	1.85
MSM39	51	4	1.51
SGRV09	16	4	3.73

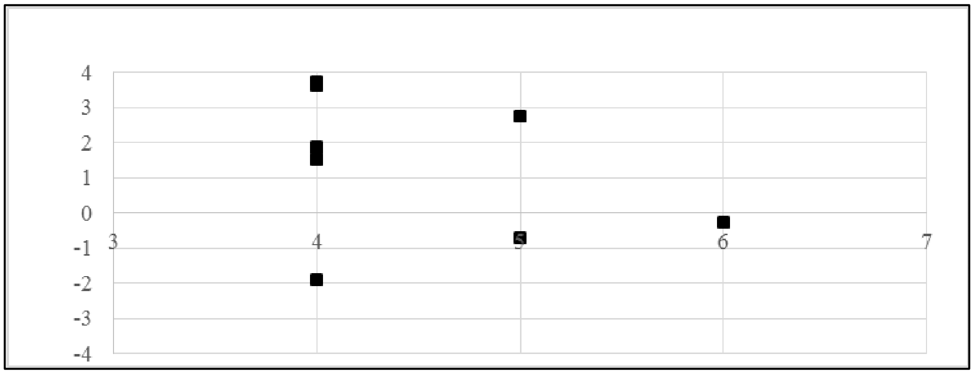

Figure 5: Scatter Plot of Final Dataset Subcontractors Participating in More than Four Projects

Findings from Final Dataset Calculation of Schedule Beta

1. Scant variation across an array of individual subcontractor activities can result in an artificially high (absolute value) Schedule Beta due to a low Variance value, thereby skewing the Schedule Beta calculation due to its use as the denominator value – i.e., Schedule Beta = Correlation / Variance.
2. Larger activity counts, in the case of the Final Dataset, in excess of one hundred (100) activities used for an individual subcontractor Schedule Beta calculation appear (anecdotally, as the project population and subcontractor contingent is not sufficient to depict a valid correlation) to yield better results.
3. Participation in a larger pool of projects, herein in the case of the Final Dataset in excess of four (4) projects appears to yield a better Schedule Beta range, but the number of subcontractors in this category is insufficient to make a correlation beyond this generalization.

Dissemination of Schedule Beta to Industry

Methodology

To properly advance the acceptance of implementation if Schedule Beta, it was decided to seek independent endorsement and standardization of the process and calculation. Much like the Experience Modification Rate (EMR) has become ubiquitous in its use and consistent in its calculation. To accomplish this, it was determined that a specification should be developed to standardize the calculation, parameters, and meaning of Schedule Beta and made available to the Construction Industry, and potentially beyond to the larger operations research realm.

Research led to the American Society of Testing and Materials (ASTM) and their respective Committee E06 on Performance of Buildings and Subcommittee E06.81 on Building Economics. Under the domain of Subcommittee E06.81 are standards and specifications such as those depicted in Table 12: Relevant Standards under the Purview of Subcommittee E06.81.

Table 12: Relevant Standards under the Purview of Subcommittee E06.81

Designation	Title
E833-14	Standard Terminology of Building Economics
E917-17	Standard Practice for Measuring Life-Cycle Costs of Buildings and Building Systems
E1699-14	Standard Practice for Performing Value Engineering (VE) / Value Analysis (VA) of Projects, Products and Processes
E2166-16	Standard Practice for Organizing and Managing Building Data
E2691-16	Standard Practice for Job Productivity Measurement

Subcommittee E06.81 develops standards to help evaluate building projects and reduce costs throughout the life cycle. Standards provide the means to compare life cycle costs of different building designs, value engineering and analysis of projects, products, and processes so that the best value may be obtained, others surround sustainability, Building Information Modeling (BIM), and other built environment innovations.

The specific remit of Subcommittee E06.81 is to provide a full spectrum of value-enabling solutions to:

1. Accelerate the adoption of new building technologies and processes,
2. Assist the evaluation of new energy efficiency and sustainability techniques, and
3. Enhance the resource productivity of labor, materials, and energy.

ASTM classifies their standards and specifications in multiple ways, the most relevant of which is the “Practice or Guide.”

- A Guide is a compendium of information or series of options that does not recommend a specific course of action. It increases the awareness of information and approaches in a given subject area.
- A Practice is a set of instructions for performing one or more specific operations that does not produce a test result. Examples of practices include: application, assessment, cleaning, collection, decontamination, inspection, installation, preparation, sampling, screening, and training.

ASTM Standard Development

This research team proposed the following to Subcommittee E06.81 for the development of a new standard for Schedule Beta:

- Type of Standard

  Anticipated as: *A Practice*

- Title of Standard

  Potentially: *"Standard Practice for Construction Project Schedule Performance Measurement"*
  "Standard Practice for Calculating a Schedule Performance Beta Index"
  "Standard Practice for Calculating a Measure of Schedule Participant Performance"
  "Standard Practice for the Comparative Measurement of Construction Subcontractor Durations"

- Proposed Scope of Standard (and Introduction)

  Suggested Introduction to the Standard:

  The Schedule Performance Measure, "Schedule Beta" measures the post-completion schedule performance – actual duration performed versus as-planned duration for their respective work – across the collective projects of a contractor or subcontractor on an ongoing basis within a defined trailing period time. Schedule Beta calculates the rolling correlation of individual project/schedule participant activities to that of the entire project for each project the entity has completed. It is an indicator of the magnitude and direction of participant performance to that of the projects. It enables owners, contractors, project managers, schedulers, estimators, supervisors and other construction project participants to compare the propensity of a contractor or subcontractor to complete their work relative to the as-planned durations and with that of the larger project. By calculating correlated deviations from planned schedule performance against that if the collective work of each respective project, a unit less index value based on the Capital Asset Pricing Model (CAPM) Beta (β) component – the measure of the volatility of an individual stock to the overall market.

  Suggested Scope:

  Based on Critical Path Method (CPM) network-based scheduling as currently used to define the relationship and durations of project activities across the construction industry, Schedule Beta provides an index measure the correlation between the deviations in as-planned duration for a participants' collective project activities across their completed projects for a defined trailing period of time versus the change from as-planned duration of the corresponding collection *of overall projects. This practice establishes a process for measuring the propensity for a contractor or subcontractor to complete their work (individual schedule activities) relative to the performance of the respective projects.*

- Keywords:

  Anticipated as: *Activity, Actual Duration, As-Planned Duration, Beta, CAPM, CPM, Critical Path Method, Duration, Performance Measure, Schedule, Schedule Beta*

- Why the Standard is Needed, How it will be Used, and By Whom (the Users)

  Suggested Justification, Use, and Users:

  The National Research Council (NRC), concluded that "[c]onstruction firms do not have a single source of metrics for comparing the efficiency of their projects and processes, or for assessing their competitive position…and that there is no single, official index or measure for the productivity of the construction industry" and that there is a "lack of an industry-wide effort to improve construction efficiency" which is caused by a "lack of [an] effective performance measures for construction-related tasks, projects and the industry as a whole." The NRC furthered in their 2009 report, Advancing the Competiveness and Efficiency of the U.S. Construction Industry, the need to "to develop effective industry-level measures for tracking the productivity of the construction industry and to enable improved efficiency and competitiveness." Schedule Beta provides the mechanism by which to address NCR's concerns and provide the efficiency-relative, competition-generating, and industry comparative measure.

  Schedule Beta will be used to track the ongoing collective performance of contractors and subcontractors relative to the completion of their work on construction projects as represented by the individual activities to that of the schedule performance of the entire project. Contractors can use this correlation to anticipate the future performance of potential subcontractors and the magnitude based upon the historical performance of the totality of work completed over a defined rolling timeframe – much in the same manner as the insurance Experience Modification Rate (EMR) or "E-Mod," which is a widely accepted three-year moving predictive ratio of actual losses to expected losses used to determine the insurance premiums that individual contractors and subcontractors must pay.

- Identify any Existing Standards and why it is Necessary to Develop an ASTM Standard

  Defined as:

  No ASTM International or other standard exists for a construction schedule performance measure/index value for comparing work across multiple projects or multiple contractors/subcontractors, or for the creation, updating, and *analysis of a critical path method (CPM) schedule network. However, a "Schedule Performance Index" (SPI) exists within the Earned Value Management (EVM) methodology.*

SPI is defined as earned value (EV) divided by planned value (PV), but is only indicative of the specific project it is measuring. It identifies the percentage of total work and cites the 'value' that a schedule or budget has achieved. SPI suffers from the problem that its values converge to 'one' (work completed) or 'zero' (work remaining) at the project end, so that the mere fact that a project was eventually finished makes a project appear successful – any problems that may have occurred are overcome, are forgotten, and become irrelevant to the final calculation and value. This gives an overly optimistic, incomplete, and non-predictive view.

It is necessary to develop a new ASTM standard to describe a comprehensive calculation method, define the meaning of the resulting Schedule Beta, and maintain the integrity of the measure.

- Identify and other ASTM Committees or Key Outside Organizations that Should be Informed

Proposed as: *The Charles Pankow Foundation,* Schedule Beta research sponsor

Construction Industry Institute (CII), Schedule Beta research co-funder

The Project Management College of Scheduling (PMCOS), [Formerly the Project Management Institute College of Scheduling (PMI-COS)], Interested party

The research team is pleased to report that Schedule Beta has been accepted by Subcommittee E06.81 on Building Economics for development as a "Practice." An initial draft version for discussion will be considered during their October 2019 meeting.

Commentary from the Industry Champion

The ability to use hard data and empirically judge a project's schedule performance has been a topic of interest to Bozzuto Building Company for quite some time. As stated in the background and introduction, the industry already uses the Experience Modification Rate (EMR) when comparing one company's safety performance to another, and so the ability to do similarly with project schedules is something of great interest to our company.

Our current methods for judging the performance of a project still involve a large amount of subjective input, which introduces flaws and therefore the possibility of skewed and incorrect results. Consistency suffers as well, as there is no common baseline to compare to from one month to the next, or between projects.

Currently I use a “stop light” report that I developed to inform our executive group on the status of each of our company’s project schedules. This report uses a subjective group of criteria that contains all the inherent flaws. Our executive group, and our company president specifically, are very interested in the results of this research study. They expect to be able to apply the results of this work to the analysis of the performance of our project schedules in real time to our current workload.

Appendix A: Behavior of Schedule Beta

Hypothetical examples have been developed to illustrate the behavior of Schedule Beta for single and multiple activities by a subcontractor across single and multiple projects.

- Case 1: A subcontractor performing two (2) activities on two (2) projects.
- Case 2: A subcontractor performing two (2) activities on one (1) project.
- Case 3: A subcontractor performing one (1) activity on two (2) projects.

Their values have been configured such that subcontractor performance tends to be late, but variation is designed such that performance includes early finish durations. Table A1: Hypothetical Schedule Beta Calculations depicts inputs and calculations for each case. Subtracting actual from as-planned durations provides deviations, "deltas," for subcontractor and project. Their means are calculated together with differences of each deviation to their mean, which are multiplied and summed, to determine the covariance. The variance squares them, as it only considers the project deviations. Dividing the covariance by the variance results in Schedule Beta for each case as follows:

- Case 1: Covariance 7.50 / Variance 25.00 = Beta 0.30
- Case 2: Covariance 0.0 / Variance 0.00 = Undefined
- Case 3: Covariance 5.00 / Variance 12.50 = 0.40.

Table A1: Hypothetical Schedule Beta Calculations

Case	Project	ID	Subcontractor			Diff. to Mean	Project			Diff. to Mean	Squared	Product of Diff.	Beta
			Actual	As Planned	Δ		Actual	As Planned	Δ				
Case 1													
1	1	1	23	20	3	1.75	110	100	10	2.5	6.25	4.375	
		2	20	19	1	-0.25	110	100	10	2.5	6.25	-0.625	
	2	A	16	15	1	-0.25	55	50	5	-2.5	6.25	0.625	
		B	15	15	0	-1.25	55	50	5	-2.5	6.25	3.125	
				Mean	**1.25**				**7.50**	**Sum**	**25.00**	**7.50**	**0.30**
Case 2													
2	1	1	23	20	3	1	110	100	10	0	0	0	
		2	20	19	1	-1	110	100	10	0	0	0	
				Mean	**2.00**				**10.00**	**Sum**	**0.00**	**0.00**	**Undefined**
Case 3													
3	1	1	23	20	3	1	110	100	10	2.50	6.25	2.50	
	2	A	16	15	1	-1	55	50	5	-2.50	6.25	2.50	
				Mean	2.00				7.50	Sum	**12.50**	**5.00**	**0.40**

These cases provide insight, explanation, and possible answers to the questions of minimum number of projects and activities required for a valid Schedule Beta calculation.

- Case 1 shows that two (2) or more projects suffice for calculating Schedule Beta.
- Case 2 shows that a single project is insufficient as data, even if the subcontractor performs multiple activities. Comparison within just one project causes a division by zero, which is undefined and mathematically illegitimate.
- Case 3 shows that a single activity from two (2) or more projects are sufficient as data.

In practice such small number of data is likely to be rare. In other words, if a comparison between multiple projects can be made, the Schedule Beta can be calculated. As expected, for the slightly tardy subcontractor, Schedule Beta is nonzero. Its order of magnitude matches its typical delay (here 1 to 3 days) with the typical project delay (here 5 to 10 days).

Another case, not considered herein as it is not an outlier, would depict a subcontractor who typically completes early relative to the project finishing late. This yields a valid Schedule Beta, which is negative (completion movement, “correlation,” in the opposite direction to that of the project). Much like the original stock market Beta with a voluminous daily history of values that are aggregated, so the schedule Beta is expected to become more robust with increased project quantity as its data.

Appendix B: Population versus Sample Size Factor – the 'N' versus 'N-1" Denominator

The fundamental statistical measure of variability around the mean of a dataset is known as the variance, as defined by:

$$\text{Variance (Population)} = \frac{1}{N}\sum(d_N - \overline{d_N})^2$$

$$\text{Variance (Sample)} = \frac{1}{N-1}\sum(d_N - \overline{d_N})^2$$

It is defined as the sum of the squared distances to the mean (squaring removes the sign, so that distances to the left and right of the mean may be aggregated). It is divided by the number of observed distances to gain the average squared distance. The square root of the variance, the standard deviation, is commonly used for convenience, because it is of the same unit as the dataset itself.

Per statistical convention, dividing by the number of observed distances distinguishes two cases:

- If the dataset is the entire population, then the full number 'N' is used as the denominator.
- If the dataset is a sample from a larger population, then "N – 1' is used as the denominator.

Variance commands in spreadsheet software (Microsoft Excel) default to the first case, because it is assumed that all data that are known to exist are included in calculating the variance. This assumption is explicitly adopted for this research, because a subcontractor should include all of their activity performance values from within the given timeframe over which Schedule Beta is calculated. Subcontractors would be discouraged from omitting any values, which could lead to manipulation of their Schedule Beta value. With the anticipated number of activities across a subcontractor's project portfolio of hundreds to thousands of values that would be part of a typical Schedule Beta, the difference between the two (2) factors – 'N' versus 'N – 1' is expected to be marginal (negligible), as the ratio N / (N – 1) approaches 1.00 and becomes "a difference without a distinction." Ultimately, the familiar analogy of 'dividing by N,' which users in practice know from calculating averages, may create a more "user-friendly" Beta formula and spreadsheet process.

Appendix C: Sensitivity of Schedule Beta – Balanced / Unbalanced Activity Splitting and Addition

Due to the two outliers noted above, a sensitivity analysis is necessary to study the behavior of Beta for the impact and/or behavior of variations in the composition and number of activities by a subcontractor within and between projects. For this purpose the following set of paired computational experiments were conceived and summarized in tabular form in Table C1: Structure of Sensitivity Experiments:

1. Splitting single activity in one project into successively smaller pieces with balanced durations
2. Splitting single activity in one project into successively smaller pieces with unbalanced durations
3. Splitting two activities in one project into successively smaller pieces with balanced durations
4. Splitting two activities in one project into successively smaller pieces with unbalanced durations
5. Splitting single activity in two projects into successively smaller pieces with balanced durations
6. Splitting single activity in two projects into successively smaller pieces with unbalanced durations
7. Adding activities to one project (i.e. expanding original scope) with delay
8. Adding activities to one project (i.e. expanding original scope) without delay
9. Adding activities to two projects (i.e. expanding original scope) with delay
10. Adding activities to two projects (i.e. expanding original scope) without delay

While a reasonable duration and level of detail for activities in network schedules is between one (1) and ten (10) workdays (i.e., two (2) workweeks),[11] during data collection from the Industry Champion it was observed that their real-world schedules are initially developed using long summary or “hammock” activities, which then – during the first several months of the ongoing project – are refined. This occurs both by being broken into smaller more detailed activities with shorter individual durations or even by adding new activities. Overall, number of activities in the schedule grows from the first baseline schedule to the last update at project completion. Since the count and duration of activities in schedules is variable, the experiments of this sensitivity analysis will inform what constitutes a proper level of detail, i.e., resolution, for a valid dataset to calculate a subcontractor’s Beta.

[11] O’Brien, J. J., Plotnick, F. L., (2015). *CPM in Construction Management*. 8th ed., McGraw-Hill Education, New York City, New York.

Table C1: Structure of Sensitivity Experiments

Experiment Number	Activities Split or Added Per Project	Projects With Splitting or Addition	Split or Addition Type
1	1	1	Balanced
2	1	1	Unbalanced
3	2	1	Balanced
4	2	1	Unbalanced
5	1	2	Balanced
6	1	2	Unbalanced
7	1	1	Delayed
8	1	1	Undelayed
9	1	2	Delayed
10	1	2	Undelayed

Table C2: Example Input for Sensitivity Analysis identifies the sample input for all experiments, which was previously studied and published[12], when analyzing how many activities are required to gain valid Beta results whatsoever. It was found that at least two (2) projects are required and at least one (1) activity per project. Otherwise, a mathematically undefined division by zero would occur due to the definition of Beta as the ratio of covariance of activity and project deltas divided by the variance of project deltas. It is expected that more data beyond this minimum will give increasingly accurate results of subcontractor performance.

Table C2: Example Input for Sensitivity Analysis[13]

Project	Activity	Subcontractor		Activity			Project		
		ID	**Description**	**Planned**	**Actual**	**Delta**	**Planned**	**Actual**	**Delta**
1	A	S1	Excavation	20	23	3	100	110	10
1	B	S1	Excavation	19	20	1	100	110	10
2	C	S1	Excavation	15	16	1	50	55	5
2	D	S1	Excavation	15	15	0	50	55	5
			Activity Count	4	Activity Variance	1.5833		Covariance	1.8750
					Project Variance	8.3333		**Beta**	**0.2250**

[12] Lucko, G., Thompson, R. C., Huynh, H. T., Su, Y. (2018). "Case Study of Project Schedule Performance Measurement Inspired by Financial Indices." *Proceedings of the 2018 Project Management College of Scheduling Annual Conference*, ed. Gorski, D. M., May 6-9, 2018, Vancouver, British Columbia, Canada, Project Management College of Scheduling, Rose Valley, Pennsylvania: 7 pp.
[13] Ibid.

If only one (1) activity was split, Activity A was selected (Experiments 1 and 2). If two (2) activities were split, Activities A and B were selected (Experiments 3 and 4) or Activities A and C (Experiments 5 and 6). If activities were added so that a multiplicity existed in one project, Activity A was selected (Experiments 7 and 8). If activities were added so that a multiplicity existed in two projects, Activities A and C were selected (Experiments 9 and 10).

Balanced splitting was performed as follows:

The initial activity duration from Table 4.7 was divided into pieces by fractions, which were then rounded to integers so that their sum always remained the same.
If one (1) day of delay had to be allocated to preserve the sum, it was given to the piece with the longest duration for consistency.

Unbalanced splitting was performed as follows:
Pieces of one (1) day planned and actual duration were split off successively from the planned or actual duration, with one piece containing the entire rest, e.g., planned 20 and actual 23 days were split unbalanced into planned 1+19 and actual 1+22 days, then into 1+1+18 and 1+1+21 days, and so forth.
Note that splitting is kept constant once all pieces reach 1 day, no shorter or fractional workdays are used.

Using the template from Table 2.9, this approach generated the complete sensitivity curves of Figures 2.1 through 2.10, where the horizontal axis is the number of pieces (see Table 2.8 for their split or addition type) and the vertical axis is Beta itself.

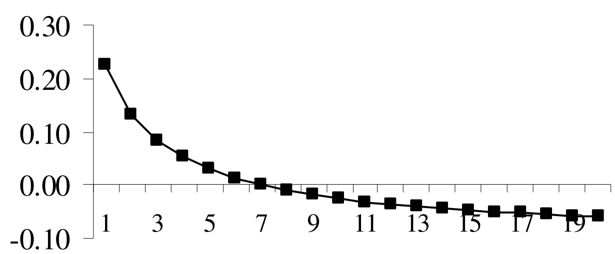

Figure C1: Experiment 1 Results

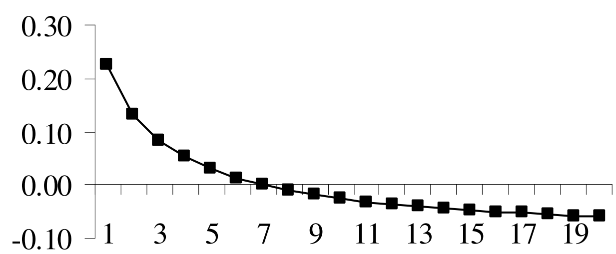

Figure C2: Experiment 2 Results

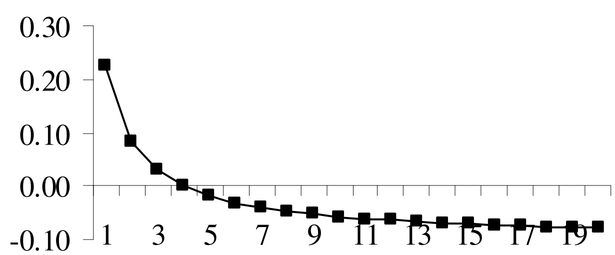

Figure C3: Experiment 3 Results

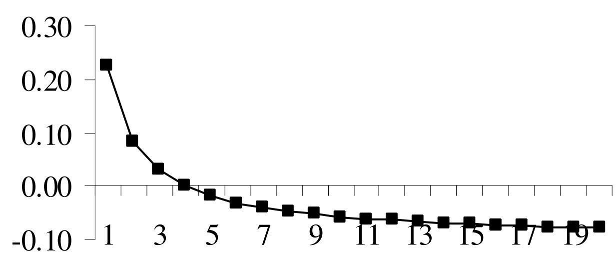

Figure C4: Experiment 4 Results

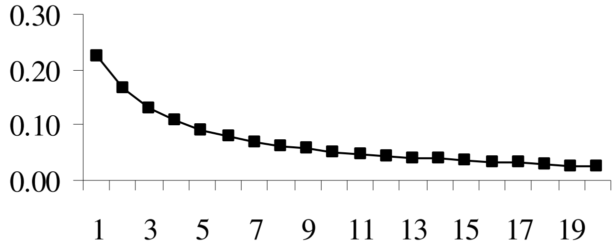

Figure C5: Experiment 5 Results

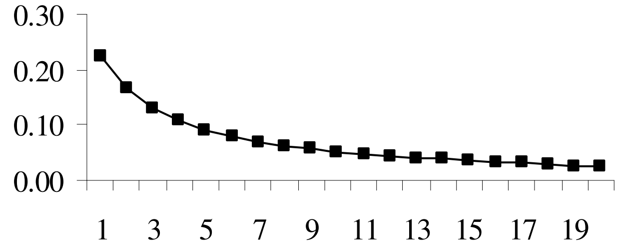

Figure C6: Experiment 6 Results

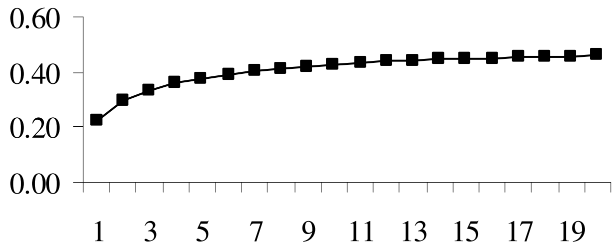

Figure C7: Experiment 7 Results

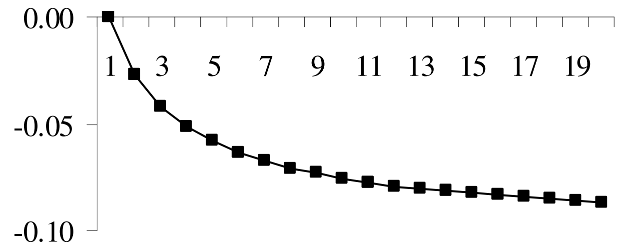

Figure C8: Experiment 8 Results

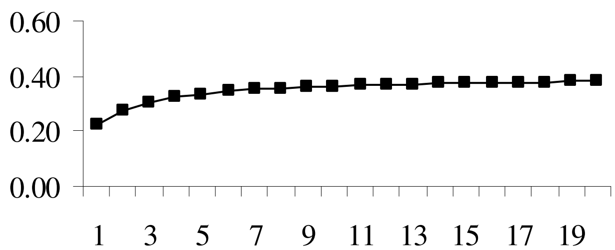

Figure C9: Experiment 9 Results

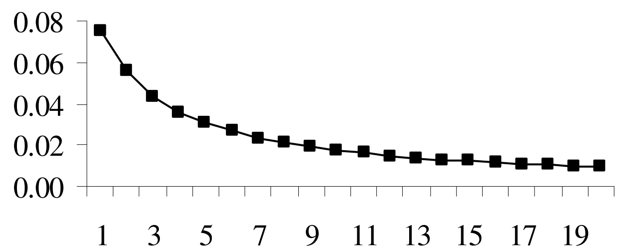

Figure C10: Experiment 10 Results

Table C3: Comparison Matrix for Eight Sensitivity Experiments summarizes the results and observations from these experiments as Figures 2.1 – 2.10 depict. All have in common a nonlinear relationship between the number of activity pieces and Beta, which is strongly asymptotic for increasing numbers of activities. Intuitively, this matches the expectation that while larger datasets will give increasingly accurate values, the improvements become ultimately negligible. For practical purposes, this is desirable, because it means that relatively small activity counts per project (single to small double digits) will suffice to gain valid Betas. The shape of the resulting curve itself reminds of the well-known curve for concrete hardening over time. Civil engineers routinely approximate it into segments, one sloped and one constant. It is suggested that the sensitivity curves be viewed in a similar light – an initial region where activity counts have a near linear influence, and a later region where activity counts become negligible.

Experiment pairs 1 and 2 and 3 and 4 give nearly identical results (with minor deviations due to rounding effects from splitting the additional Activity B in a similar manner. This means that Beta is not sensitive to where splitting occurs. Beta becomes not just smaller, but even slightly negative for larger numbers of activity pieces. It underlines the importance of selecting a properly sized dataset to be routinely used – excessive splitting as well as having only a single data point are statistically undesirable. They should be discouraged because they could lead to manipulation of Beta.

The paired Experiments 1 and 2 and 3 and 4 with 5 and 6 added give exactly identical results. This means that any balanced or unbalanced split within a single or multiple activities has no impact on Beta. This is intuitive, because the sum of the deltas is unchanged. For practical purposes, this is desirable, because it means that Beta will be unaffected by different approaches of activity splitting.

Interestingly, the behavior of Beta is inverted for Experiments 7 and 9. These add additional activities to a project, thus expanding its scope. Since each of them is delayed, they will collective make Beta increasingly worse (i.e., stronger correlation with the project that is assumed to be delayed itself). This is the reason for the worsening, i.e., growing, value of Beta is confirmed by Experiments 8 and 10. Here the additional activities are not delayed, which has the opposite effect of lowering Beta consistently and acting opposite to the project that is assumed to be delayed, so that all Beta values are purely negative.

Table C3: Comparison Matrix for Eight Sensitivity Experiments

Experiment Number	Curvature	Shape	Range
1	Concave	Asymptotic	Positive to negative
2	Concave	Asymptotic	Positive to negative
3	Concave	Asymptotic	Positive to negative
4	Concave	Asymptotic	Positive to negative
5	Concave	Asymptotic	Positive
6	Concave	Asymptotic	Positive
7	Convex	Asymptotic	Positive
8	Concave	Asymptotic	Negative
9	Convex	Asymptotic	Positive
10	Concave	Asymptotic	Negative